LESSON PLAN: SNAP, CRACKLE, POP: Submarine Buoyancy, Compression, and Rotational Equilibrium
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2012 NAVAL HISTORICAL FOUNDATION TEACHER FELLOWSHIP

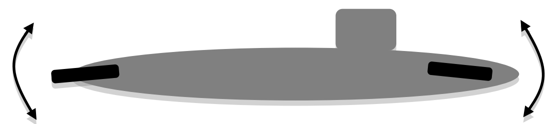
ACTIVITY 1: Buoyancy Problems

OBJECTIVE: Practice and Reinforce concepts related to Fluid Pressure, primarily Buoyancy

MATERIALS: How A Submarine Submerges - Video This clip "Buoyancy: Take 'Er Down" and many others are available on the US Navy History Museum's You Tube Channel

INSTRUCTIONS:

- Submarines are basically teardrop shaped, with a "sail": a rectangular protrusion on the top, from which the periscopes and antennas emerge. The image depicts a submarine that is moving to the right.
- Submarines have two sets of "planes" that can be angled so that the water flowing past them pushes them up or down. In this way, they are used to control the forward/aft angle of the submarine (its pitch). Similar to the wing flaps of an airplane, they are also used to change forward motion into vertical motion. One set is near the rear (aft) end of the submarine, or its stern. The other is either near the front (bow), or protruding from the sail.



- Two ways to control the vertical position of a submarine:
    - adjust its own density and thus the net vertical force it experiences by pumping water from its tanks into the ocean, or allowing some ocean water into its tanks.
    - changing some of its horizontal motion into vertical motion by angling its "planes".
- Does the buoyant force on an object change as the object is taken deeper in a fluid?
    - Normally we assume it does not, since the buoyant force is determined by how much fluid the object displaces, which is determined by the object's volume and the density of the fluid, which we often assume to be constant, but…
    - If and object is made of a compressible material, as it goes deeper into the fluid, it undergoes what we call Bulk Deformation, such that it takes up less space (occupies a smaller volume), so Yes, it experiences a weaker buoyant force as its volume decreases. A submarine's hull actually compresses (yes, it gets measurably smaller! …not noticeably smaller, but measurably) due to the water pressure it experiences. Activity 3 of this lesson plan explores this concept farther.
        - To prove this, some crews have tied a string tightly between the bulkheads (walls) while on surface. After diving to a deep depth, the string hangs loosely!
    - Also, although liquids are basically incompressible (you cannot make a defined amount of liquid take up less space by squeezing it, AKA its density does not change as you increase the pressure on it). Nevertheless, if the pressure is extreme, you can vary the density of a liquid.
    - Although pressure does have a small effect on the density of water, temperature has a larger effect, and the temperature of seawater drops sharply as you go deeper. Overall, the compression of the hull has a larger effect than either of these effects on the density of the seawater.

density of air = 1.25 kg/m^3

density of seawater = 1025 kg/m^3

1m = 3.281 feet

volume of submarine = 8650 m^3

air pressure at sea level = 1ATM = or 1.013×10^5 Pa

1. A) What water pressure would a submarine experience when it is 400 feet below the surface of the ocean?

Recall that oceans consist of saltwater... ☺

- Convert 400 ft to meters: 400 ft (1m / 3.281 ft) = 121.914m
- Difference in Fluid Pressure = density of fluid * g * difference in depth
- $\Delta P_{\text{due to layers of water above}}$ = 1025kg/m^3 * 9.8m/s^2 * 121.914m = 1224626 Pascals

B) What is the absolute pressure that the submarine experiences at this location (combination of the water pressure with the pressure caused by the air above the water)?

- Air pressure at sea level = 1 ATM = 1.013×10^5 Pa
- Absolute pressure (due to water And air) = 1224626 Pascals + 1.013×10^5 Pa
- Absolute Pressure = 1325926 Pa

C) What percent of the absolute pressure at 400 ft is contributed by the miles of air in our atmosphere? #1 C)

- While underneath merely 400 feet of ocean water, the contribution of air pressure from the miles of air above the ocean is less than 10%... 1.013×10^5 Pa / 1.326×10^6 Pa = .076 or 7.6%

2. A) What is the difference in air pressure when an airplane increases its altitude by 400 feet?

- $\Delta P_{\text{due to layers of fluid}}$ = density of fluid * g * difference in altitude
- $\Delta P_{\text{due to layers of fluid}}$ = 1.25kg/m^3 * 9.8m/s^2 * 121.914m
- $\Delta P_{\text{due to changing altitude}}$ by 400 feet = 1493.44 Pa

B) Is this a significant percentage of regular atmospheric pressure, such that a human would notice or be affected? Calculate it and comment.

- 1493.44 Pa / 1.013×10^5 Pa = .0147 or 1.5%
- A human would barely notice this.

3. A) Some Navy aircraft, like the F-35 Joint Strike Fighter shown, can fly at altitudes over 40,000 ft. To understand why pilots of the F-35 need to have an oxygen system to help them

breathe, calculate the air pressure at 10,000 ft, where many commercial aircraft fly.

- Convert 10000 ft to meters: 10000 ft (1m / 3.281 ft) = 3047.9m
- $\Delta P_{\text{due to layers of fluid}}$ = density of fluid * g * difference in altitude
- $\Delta P_{\text{due to layers of air}}$ = 1.25kg/m^3 * 9.8m/s^2 * 3047.9m
- $\Delta P_{\text{due to changing altitude}}$ by 10000 feet = 37336.2 Pa
- P at 10,000 ft = $P_{\text{sea Level}}$ - $\Delta P_{\text{due to changing altitude}}$ by 10000 feet = 1.013x10^5 -37336.2 = 63963.8 Pa

B) What percentage of sea level air pressure is this?

- 63963.8 Pa / 1.013x10^5 Pa = .6314 or 63%
- Note: This is only a very rough estimate, since as you increase in altitude, the density of the air decreases significantly from the sea level value of 1.25kg/m^3. Thus, the decrease would be less than we calculated. The air pressure at 10,000 ft is closer to 70,000 Pa.

4. How much water would you need to be under, such that the absolute pressure (air and water pressure) is double the regular atmospheric air pressure you experience at sea level?

- $\Delta P_{\text{due to layers of water}}$ = Regular atmospheric air pressure = 1.013x10^5 Pa.
- $\Delta P_{\text{due to layers of fluid}}$ = density of fluid * g * difference in depth
- 1.013x10^5 Pa= 1025kg/m^3 * 9.8m/s^2 * depth
- depth = 10.1m or ~ 33.1 ft

5. A submarine has a mass of 6.97million kg and does not change its mass by adding or removing any water during this problem.
   Assume that:
   - The density of seawater is 1025 kg/m^3 and does not change appreciably with depth or temperature.
   - The volume of the submarine at the surface is 6758m^3
   - The volume of the compressed submarine at 800ft is 6400m^3

   A) When it is completely submerged just below the surface, calculate its density and its vertical acceleration, based solely on its weight and the buoyant force it is experiencing. Ignore any forces exerted by the water on the planes. Your solution should include a free body diagram.

Just below the surface:

- Density = Mass / Volume
- Weight = mass * acceleration due to gravity
- B = weight of displaced fluid
- Vertical Acceleration of sub = Net Vertical Force exerted on the sub / $mass_{submarine}$
- Accel. = B - $W_{submarine}$ / 6.97 E 6
  = (6.79E7 – 6.83E7) / 6.97 E 6
  = -.0574 m/s^2 or .06 m/s^2 downward

Quantity	Submarine	Displaced Fluid (seawater)
Weight (N)	6.831×10^7	6.79 E 7
Mass (kg)	6.97×10^6	6.93 E 6
Volume (m^3)	6758	6758
Density (kg/m^3)	1031	1025

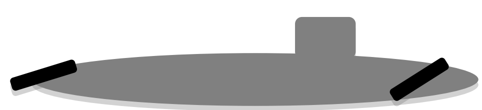
   B) When it is at 800ft, calculate:
      i. the density of the submarine
      ii. the buoyant force exerted on the submarine
      iii. the sub's new vertical acceleration.
      - Your solution should include a free body diagram with arrows drawn to the same scale as those in part A.

At 800 ft

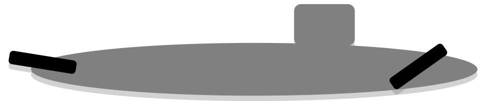
- Vert. $accel._{sub} = \Sigma F_{\text{on the sub}} / m_{sub}$
- Accel. = B - $W_{submarine}$ / 6.97 E 6
  = (6.43E7 – 6.83E7) / 6.97 E 6
  = -.574 m/s^2 or .6 m/s^2 downward

Quantity	Submarine	Displaced Fluid (seawater)
Weight (N)	6.831×10^7	6.43 E 7
Mass (kg)	6.97×10^6	6.56 E 6
Volume (m^3)	6400	6400
Density (kg/m^3)	1089	1025

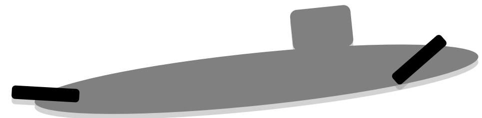
   C) Draw a simple submarine with its planes in positions that you think would help maintain its depth in this situation. Explain your choice of positions.



This would be acceptable, if explained that as water collided with each set of planes, it would exert upward forces on both of them, creating zero net torque, but an overall upward force that would be able to overcome the problem that the weight of the submarine is presently greater than the buoyant force it is experiencing. Notice that the angle on the bow planes is greater than the angle on the stern planes, since the bow planes are closer to the center of gravity of the sub and thus its axis of rotation. The stern planes have a longer lever arm, so we do not need as strong a force on them, or the submarine would begin to point downward. Staying level is preferred, unless using the angle of the entire ship to help change depth.



…and this, although a more aggressive solution and only necessary if the downward acceleration were excessive, would be acceptable, since it would cause a counter clockwise torque on the submarine, causing the entire sub to point toward the surface, making the entire bottom of the sub into a “plane” type surface that water would hit and create an upward force that could support the sub, keeping it from continuing to accelerate downward.



This would be a temporary situation, held only until the submarine achieved an appropriate upward angle. Notice that the stern planes are only angled downward a little bit, since they have a long lever arm and do not need to exert as strong a force to have an affect on the angle of the sub.

Air Pressure: Conversion / Volume of a Cylinder / Pressure vs. Volume

6. A) When the submarine compresses due to the high water pressure, does the air pressure in the sub increase?

The bottom line is Yes, the air pressure does increase, but only a little bit, such that it would be barely noticeable…

- This is because gases are compressible, whereas solids and for the most part, liquids are not

B) If so, by how much,

You may assume the following:

- When the submarine is at the surface, the air inside it is at sea level atmospheric pressure
- Recall that PV=nRT
  - assume n (#molecules), R(gas constant), T(Temperature) are constant.
- The submarine is a simple cylinder.
- radius at the surface (before compression) = 198inches
- radius after compression = 197inches
- uncompressed length = 360 feet
- compressed length = 359 feet 9 inches

First determine how much the volume of the submarine changes.

- delta radius = 1in (.025399m) out of 198in (5.02895m)
- delta length = 3in (.076196m) out of 4320in (360feet, 109.7226m)
- initial volume = $\pi r^2 h$ = 8717.67m^3
- compressed volume = 8623.8447m^3
- delta volume = 93.825m^3 Note, this delta Volume is not used in the rest of the problem.

Then, applying the general rule for how the pressure of a sample of gas is related to its temperature and pressure…

- PV=nRT assuming n, T are constant, we can say that P = constant (1 / V)
- $P_{original}$ = constant ($1/V_{original}$)
- 101300 = constant (1/8717.67)
- constant = 883099971

- P_{final} = constant ($1/V_{final}$)
- P_{final} = 883099971 (1/ 8623.8447)
- P_{final} = 102402
- delta p = 1102Pa

and would it be noticeable?

- Difference in Fluid Pressure = density * g * h
- 1102 Pa = 1000 (9.8) h
- h = .11m or 11cm
  - the air pressure change would be about the same as going under 11cm of water, which a human could barely notice.