Teacher Resources

How Linear are Submarine Missiles?

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Introduction:

In this unit of study students will relate the dimensions and characteristics of submarine ballistic missiles to scatter plots, volume calculations, and linear regression analysis. Students begin the unit by discussing background information on U.S. Navy Fleet Ballistic Missiles and then completing a group activity to build a life size missile model to understand the immense size of these missiles carried in U.S. Navy ballistic missile submarines. Students then read and evaluate a missile data scatter-plot. Next, students calculate missile volumes from given missile dimensions. Finally, students use their data to compare missile volume to the distance a missile travels, determining the relationship between volume and distance.

Common Core Math Standards

High School

Domain: Interpreting Categorical and Quantitative Data ***S-ID***

Standard: Summarize, represent and interpret data on two categorical and quantitative variables.

Cluster: 6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.
b. Informally assess the fit of a function by plotting and analyzing residuals.
c. Fit a linear function for a scatter plot that suggest linear association.

Standard: Interpret linear models

Cluster: 7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Domain: Geometry Measurement and Dimensions ***G-GMD***

Standard: Explain volume formulas and use them to solve problems.

Cluster: 3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Domain: Building Functions ***F-BF***

Standard: Build a function that models a relationship between two quantities

Cluster: 1. Write a function that describes a relationship between two quantities.

Middle School

Domain: Geometry ***8.G***

Standard: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Cluster: 9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Domain: Statistics and Probability ***8.SP***

Standard: Investigate patterns of association in bivariate data.

Cluster: 1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association and nonlinear association.

Cluster: 2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

Cluster: 3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

Domain: Functions ***8.F***

Standard: Use functions to model relationships between quantities.

Cluster: 4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x,y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values.

Cluster: 5. Describe qualitatively the functional relationship between two quantities by analyzing a graph.

Instructional Procedures for Activities:

- ***Introduction to submarine missiles – "Background on Submarine Ballistic Missiles" with embedded video links to historical submarine ballistic missile launches:***

The U.S. Navy ballistic missile submarines also called "boomers" carried various types of Fleet Ballistic Missiles (FBM's) during the Cold War (see figure below). The missiles evolved from the single warhead Polaris A1 and A2 to the multiple warhead Polaris A3 through the independently targetable multiple warhead Poseidon A3 and Trident I C4 to the current Trident II D5 missile. The progression from a single warhead to multiple warheads to independently targetable warheads provided increased target coverage and required larger missiles.

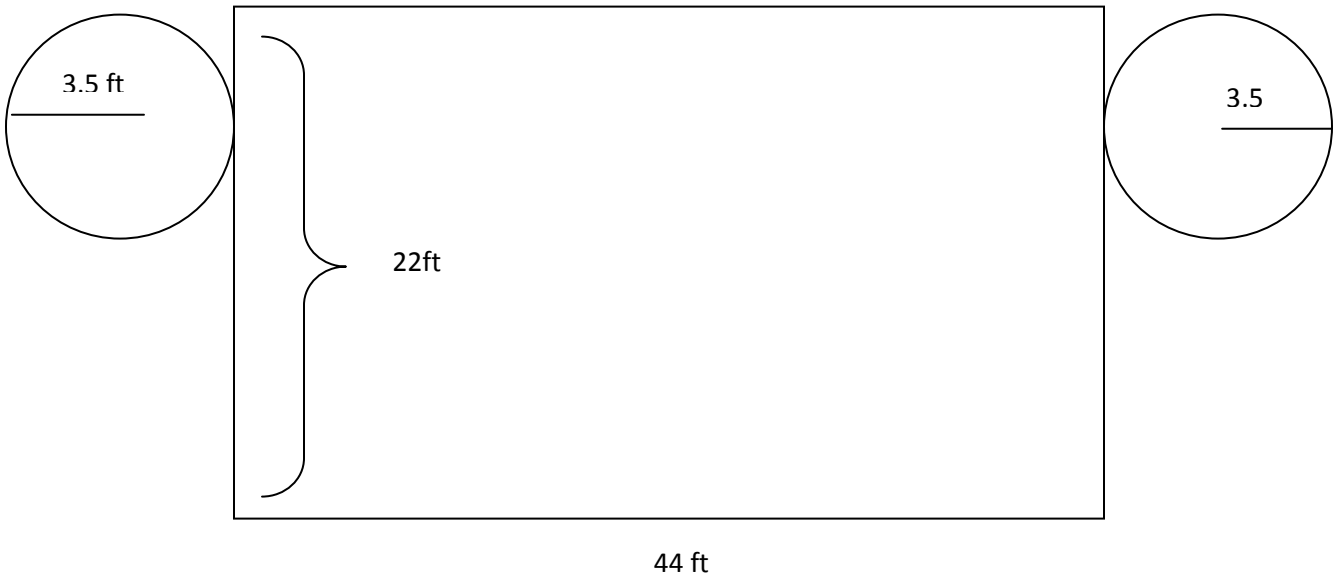
- *Polaris A1, named for Polaris, the North Star, was a 2-stage ballistic missile, powered by solid fuel rocket motors. The first successful underwater launching of a Polaris A1 was conducted in 1960 and was officially retired from active duty in 1965. It had a 1380 mile range.*
- *The Polaris A2 was similar to the A1 except it was longer allowing it to carry more fuel which made it heavier and increasing the range to 1730 miles. The A2 was first launched in 1961 and was officially retired in 1974.*
- *The Polaris A3 missile with multiple warheads represented a significantly greater technological advancement over A2 and A1. The first successful launch of the A3 was in 1963. Its longer range of 2880 miles meant that no land target was beyond reach. The A3 missile was retired in 1982.*
- *The Poseidon C3, named after the mythological Greek god of the sea, had its roots in Polaris technology. It increased in size again, with a 2-foot longer length and 20-inch greater diameter. Its multiple warheads, each of which could be targeted separately, improved the effectiveness of the FBM weapons system as a deterrent to the outbreak of a nuclear war. The C3's first successful launching was conducted in 1970. The C3 missile was finally retired in 1992, after the Cold War's end.*
- *The Trident I C4 missile name stems from roman mythology. The Trident I C4 missile is a 3-stage, solid propellant missile. Its advanced propellant provides a 4600 mile range. The Trident 1 C4 missile was test launched in 1977 and was retired in 2005.*
- *The Trident II D5 missile is also a 3-stage, solid propellant missile. The Trident II is more sophisticated and larger than its predecessor Trident I: 10 feet longer and 9 inches greater in diameter. The Trident II D5 was first launched in 1989 near the end of the Cold War. D5 is the only fleet ballistic missile loaded aboard our 14 Trident-class ballistic missile submarines today.*
- *The purpose of nuclear-warhead tipped fleet ballistic missiles is to deter a nuclear war, and respond if necessary.*

- ***Hook Activity – See "Blow-Up a Missile" activity and Teacher Notes below***
- ***Activity 1 – See "OMG! Missiles are Scattered Everywhere" and Teacher Notes below***
- ***Activity 2 – See "Massive Missile Insides" and Teacher Notes below***
- ***Activity 3 – See "How Far will a Missile Go?" and Teacher Notes below***

Hook Activity: Blow-Up a Missile!

Teacher Notes

Discuss as a class the information needed to find the dimensions of the cylinder to build the missile. Discuss the shape as being a cylinder and the 2 dimensional shapes the make a cylinder (rectangle and 2 circles). Using the dimensions: diameter 84 inches and length 44 feet, discuss the amount of painters plastic needed to make each part of the cylinder.



Group 1 (upper level group)- circles

Group 2 (lower level group) – rectangle 12X22

Group 3 (lower level group) – rectangle 12X22

Group 4 (lower level group) – rectangle 12X22

} these groups could be combined, if needed

Group 5 (middle level group) – rectangle 8X22

Activity 1 - OMG! Missiles are Scattered Everywhere

Teacher Notes

Goals To assess students'

- *Skill in identifying the 2 values associated with each point in a scatter plot.*
- *Ability to interpret trends in data in a scatter plot*
- *Ability to use trends to make predictions.*

Materials A copy of the black line master "Reading a Missile Scatter Plot" for each student

Activity

Discuss the information/background on missiles. Distribute a copy of the activity sheet to each student. The students should work with a partner to help enrich their interpretation of the data. As you observe the pairs working, encourage them to give clear explanations of the reasoning behind their answers.

Discussion

Questions 1 and 2 familiarize students with reading data points on a scatter plot by asking them to read information directly from the graph. For Question 1, the students need to count the number of dots above the 54 on the horizontal axis. For question 2, the students have to interpret the two values associated with point A.

Questions 3, 4 and 5 ask the students to make predictions or interpolations from the data. Questions 3 and 4 ask for an interpolation within the range of values of the data and question 5 asks for predictions that go beyond the range of the data.

Through oral discussions or the student's written explanation, you should be able to determine whether the students have some sense of the linear trend of the data.
That is, their predictions should not be just wild guesses; they should be supported by arguments.

Question 6 assesses the students' ability to describe the relationship between the two variables. The point of the question is not to elicit an exact answer. Rather, the responses will allow you to assess whether the students can use their observations logically to formulate a description of the relationship that make sense (both to you and to other students).

Solutions

1. *3 missiles. Count the number of dots above 54 on the horizontal (diameter) axis.*
2. *Point A represents a weight of 73,000 pounds and a diameter of 74 inches. These values are determined by reading the y-coordinate and the x-coordinate.*
3. *About 80 inches. Imagine a line going through the data and then find the horizontal coordinate of the point associated with 100,000 pounds.*
4. *About 52,000 pounds. Imaging a line going through the data and then find the y coordinate of the point associated with 65 inches in diameter.*
5. *About 98 inches in diameter. Answers may vary. Except all if reasonable, not just guesses.*

As the diameter increases, the weight increases so the relationship is nearly linear.

Activity 2 - Massive Missile Insides

Teacher Notes

Calculations for Missile Volumes

Using the formula for the volume of a cylinder $\boldsymbol{V = \pi r^2 h}$, these calculations were made.

Polaris A-1 volume of stages 1 and 2 combined. $V = \pi(2.25)^2(14) = \mathbf{70.875\pi}$

Polaris A-2 volume of stages 1 and 2 combined. $V = \pi(2.25)^2(17) = \mathbf{86.0625\pi}$

Polaris A-3 volume of stages 1 and 2 combined. $V = \pi(2.25)^2(18) = \mathbf{91.125\pi}$

Poseidon C-3 volume of stages 1 and 2 combined. $V = \pi(3.08)^2(20) = \mathbf{189.728\pi}$

volume of stage 3 $V = \pi(2.06)^2(8) = \mathbf{33.9488\pi}$

volume of stages 1, 2 and 3 combined. $V = 189.728\pi + 33.9488\pi = \mathbf{223.6768\pi}$

Trident I C-4 volume of stages 1 and 2 combined. $V = \pi(3.08)^2(20) = \mathbf{189.728\pi}$

volume of stage 3 $V = \pi(2.06)^2(8) = \mathbf{33.9488\pi}$

volume of stages 1, 2 and 3 combined. $V = 189.728\pi + 33.9488\pi = \mathbf{223.6768\pi}$

Trident II D-5 volume of stages 1 and 2 combined. $V = \pi(3.5)^2(29) = \mathbf{355.25\pi}$

volume of stage 3 $V = \pi(2.33)^2(8) = \mathbf{43.4312\pi}$

volume of stages 1, 2 and 3 combined. $V = 355.25\pi + 43.4312\pi = \mathbf{398.6812\pi}$

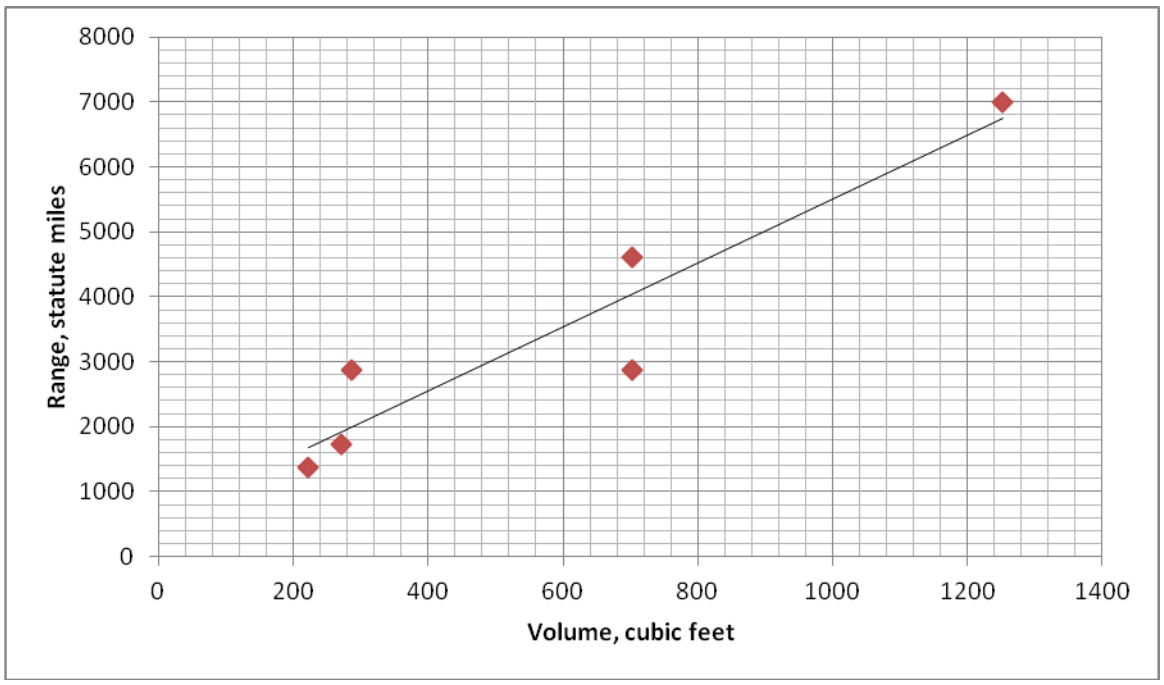
***Remember that in order to find the 3rd stage radius, first divide the diameters by 3, then by 2.**

Activity 3 - How Far Will A Missile Go?

Teacher Notes

1. Using Grid paper make a scatter plot to display the relationship between total missile volume and the range of the missile. Describe the relationship you see.

   Positive relationship



2. Draw a line through the points on the graph. Use the line to find the equation for the line of fit. Write the equation for the line in slope-intercept form.

   $y = 4.921x + 592.652$

   (This is the calculator generated equation. The equations that are configured by hand may be close but probably not exact.)

3. Use the graph of the line and the equation of the line to identify the slope and the y-intercept. Explain what these values tell about the volume and the range of the missiles.

   The slope is 4.921 and the y-intercept is 592.652.

The slope tells me how much the volume changes with the addition of each mile. The y-intercept tells me what the range would be if the volume of the missile is 0, which does not make sense in this real world situation.

4. Use your equation to find the range of the missile that has a volume of 500 cubic feet.

   ~3053 miles

5. Use your equation to find the range of the missile that has a volume of 1500 cubic feet.

   ~7973.8 miles

6. Use your equation to find the volume of a missile that traveled a range of 3500 miles.

   ~591 cubic feet

7. Use your equation to find the volume of a missile that traveled a range of 800 miles.

   ~43 cubic feet

8. ***Using Technology.*** *Using the graphing calculator enter the data points into the list menu.*
   A. Use the calculator to determine the equation that will best describe your data. Compare it to the equation you developed in question 2.

   B. Using the table set and table key options on the calculator, find the range for a volume 500 cubic feet and 1500 cubic feet.

   C. Using the table key option on the calculator. Find the volume of a missile that traveled 3500 miles; and 800 miles.